ԵՐԵՎԱՆԻ ՊԵՏԱԿԱՆ ՀԱՄԱԼՍԱՐԱՆ

Վիգեն Հայկի Խալաթյան

Սահմանափակ ուռուցիկ մարմինների վերականգնումը տոմոգրաֆիկ եղանակներով

Ա.01.05 «Հավանականությունների տեսություն և մաթեմատիկական վիճակագրություն» մասնագիտությամբ ֆիզիկամաթեմատիկական գիտությունների թեկնածուի գիտական աստիճանի հայցման ատենախոսության

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Reconstruction of bounded convex bodies using tomographic methods

ABSTRACT

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Ատենախոսության թեման հաստատվել է Երևանի պետական համալսարանում:

Գիտական ղեկավար՝	ֆիզմաթ. գիտ. դոկտոր Վ.Կ. Օհանյան
Պաշտոնական ընդդիմախոսներ՝	ֆիզմաթ. գիտ. դոկտոր Ռ.Հ. Արամյան
	ֆիզմաթ. գիտ. թեկնածու Ա. Ռ. Սիմոնյան
Առաջատար կազմակերպություն՝	ՀՀ ԳԱԱ ինֆորմատիկայի և ավտոմատացման պրոբլեմների ինստիտուտ

Պաշտպանությունը կկայանա 2022 թ. հուլիսի 27-ին՝ ժամը 12:00-ին, Երևանի պետական համալսարանում գործող ԲՈԿ-ի 050 մասնագիտական խորհրդի նիստում (0025, Երևան, Ալեք Մանուկյան 1):

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The topic of dissertation was approved by the Yerevan State University

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Leading organization:	Institute for informatics and automation problems of NAS RA

Dissertation defense will take place on 27th of July, 2022 at 12:00, at the meeting of the Specialized Council 050 of the Higher Attestation Commission at YSU (1 Alex Manoogian, Yeravan 0025, Armenia).

The dissertation is available in the library of Yerevan State University. The abstract of dissertation was distributed on 15th of June, 2022.

Scientific secretary of specialized council T. N. Harutyunyan

General description

Topicality: In the last century German mathematician W. Blaschke (see [1]) proposed the problem of finding necessary and sufficient conditions for a sequence of real numbers to be the sequence associated with a convex set and of determining when those conditions are satisfied, whether or not the corresponding convex set is unique. In particular, Blaschke problem was formulized of recognition of bounded convex bodies using covariogram, orientation-dependent chord length distribution function, chord length distribution function etc.

Matheron in 1986 asked the following question:

Covariogram Problem. Does the covariogram determine a convex body, among all convex bodies, up to translations and reflections?

The first partial solution of Matheron's conjecture gave Nagel in 1993 (see [5]). Nagel proved that every planar convex polygon is determined within all planar convex polygons by its covariogram, up to translations and reflections. In 2002 Bianchi, Segala and Volcic provided that if the planar convex body's boundary has strictly positive continuous curvature the covariogram problem has positive answer (see [6]). In 2009 G. Averkov and G. Bianchi showed that every planar convex body is determined within all planar convex bodies by its covariogram, up to translations and reflections (see [9]). In [7] Bianchi provided that if D $\subset R^n$ and H $\subset R^n$ be convex bodies and let L be a nondegenerate linear transformation, then the two convex bodies L(K $\times$ H) and L(K $\times$(-H)) in R^{n+k} have the same covariogram. This means that the covariogram problem in the general case ($n \geq 4$) has a negative answer.

There is another problem which formulated as such a way.

Blaschke problem Is there a one-to-one correspondence between bounded convex bodies D and their chord length distribution functions $F_D(t)$?

This was an old question of W. Blaschke whether the random chord length determines the convex body D uniquely, up to all Euclidean motions. The answer of this question was given by Mallow and Clark (see [16]), they construct two non-congruent bounded convex 12-gons with the same chord length distribution. It means that we cannot reconstruct convex bodies by

Method to avoid the above mentioned problem is not using completely mixed form of the distribution, but preserving information about the line which generates the chord (see [21], [5]) and to examine the above mentioned question in subclasses of the class of convex bodies for which the chord length distribution provides sufficient information to distinguish between non-congruent members (see [17], [18]).

The covariogram problem appears independently in other contexts. Adler and Pyke (see [30], [31]) asked whether the distribution of the difference P_1 - P_2 of two independent random points P_1, P_2 uniformly distributed over D determines D up to translations and reflections. This problem is equivalent to the covariogram problem, from the observation that the convolution in (0.1) is the probability density of P_1 - P_2 , up to a multiplicative factor.

It is very little known about the above mentioned problem. One of the result of this dissertation is the explicit form of relationship between covariogram and distribution function of the distance between two uniformly and independently distributed points which is presented in second chapter.

Objectives:

1) Explicit form of kinematic measure in terms of chord length distribution function

2)Relationship between covariogram and distribution function of the distance between two uniformly and independently points

3)The calculation of the covariogram and Orientation-dependent chord length distribution function for any trapezoid.

4)Relationships between the covariogram and the orientation-dependent chord length distribution function of an oblique prism and those of its base.

5) Explicit forms of the covariogram and the orientation-dependent chord length distribution function of an oblique prism with circular, elliptical, trapezoidal and triangular bases.

Research methods: Methods from Integral and Stochastic Geometry and Probability Theory.

Main results of the dissertation are published in four papers; references can be found in the end of this booklet.

Structure and volume of dissertation: the dissertation is written on 76 pages; consists of an introduction, three chapters, conclusion and the list of 41 cited references.

Overview and main results

Let R^n (n $\geq$ 2) be the n-dimensional Euclidean space, $D \subset R^n$ be a bounded convex body with inner points and V_n be n-dimensional Lebesgue measure in R^n. The covariogram $C(D, h)$ of D is the function (see [2])

$$C(D, h) = V_n(D \cap (D + h)),$$

where $h \in R^n$ and $D + h = \{y + h, y \in D\}$ is the translation of D by h. Another way of representing covariogram is autocorrelation of the characteristic function of D:

$$C(D, h) = I_D * I_{(-D)} = \int_{R^n} I_D(t) * I_{(-D)}(h - t)dt \qquad (0.1)$$

where $*$ denotes convolution and -D is the reflection of D in the origin.

Denote by G the space of all lines g in the Euclidean plane R^2. The space G of lines can be mapped in a one-to-one way onto the surface of a unit cylinder with the usual cylindrical coordinates (p, ϕ), where ϕ is the direction of g (ϕ is a point on the unit circle) and p is the distance from the origin O to g. Let $\nu(\cdot)$ be the locally finite measure on G, invariant with respect to the group of all Euclidean motions (translations and rotations).

It is well known that the element of the measure up to a constant factor has the following form (see [13]):

$$\nu(dg) = dg = dp d\phi$$

where dp is the one-dimensional Lebesgue measure and dϕ is the uniform measure on the unit circle.

We denote by [D] all lines that intersect bounded convex body D

$$[D] = \{g \in G : g \cap D \neq \emptyset\},$$

Furthermore, let A_D^t be the set of lines that intersect D, producing a chord χ(g) = g $\cap$ D of length less than y, i.e.

$$A_D^t = \{g \in [D] : |\chi(g)| \leq t\}, \quad t \in R.$$

The distribution function for the length of a random chord χ of D is usually defined as

$$F_D(t) = \frac{\nu(A_D^t)}{|\partial D|} = \frac{1}{|\partial D|} \int \int_{A_D^t} dp d\phi$$

the same way we can define chord length distribution function for higher dimensional spaces[14].

Let P_1 and P_2 be two points chosen at random, independently and with uniform distribution in D. Firstly, we are going to find the probability that the distance $\rho(P_1, P_2)$ between P_1 and P_2 is less or equal to x, that is we would like to find the distribution function $F_\rho(x)$ of $\rho(P_1, P_2)$. By definition, we have

$$F_\rho(t) = P(P_1, P_2 \in D : \rho(P_1, P_2) \leq t) = \frac{\iint\limits_{\{(P_1, P_2)\,:\,\rho(P_1, P_2) \leq t\}} dP_1 dP_2}{\iint\limits_{\{P_1, P_2 \in D\}} dP_1 dP_2}$$

where dP_i, $i = 1, 2$ is an element of Lebesgue measure in R^n.

Let O_n be the surface area of the n-dimensional unit sphere. O_n is defined [1]

$$O_n = \frac{2\pi^{(n+1)/2}}{\Gamma(\frac{n+1}{2})}$$

where Γ is the gamma function which satisfies the recursion formula,

$$\Gamma(n+1) = n\Gamma(n)$$

especially, $\Gamma(n+1) = n!$ and $\Gamma(1/2) = \sqrt{\pi}$.

Theorem 1. *For any body D in R^n*

$$K(D,t) = \frac{O_{n-1}}{2} V(D) - \frac{O_{n-2}}{2(n-1)} S(D) \int_0^t [1 - F_D(u)] du \tag{0.2}$$

where $K(D,t)$ is the kinematic measure of all oriented segments of length t that lie inside D

We know that

$$f_\rho(t) = \frac{t^{n-1} K(D,x)}{V^2(D)} \tag{0.3}$$

Using equations (0.2) and (0.3) we get the relationship between the density function of the distance between two points chosen at random, independently, and with uniform distribution in D and the chord length distribution function as follows

$$f_\rho(t) = \frac{1}{V^2(D)} \Big(t^{n-1} O_{n-1} V(D) - \frac{t^{n-1} O_{n-2}}{(n-1)} S(D) \int_0^t [1 - F_D(u)] du \Big). \tag{0.4}$$

From equation (0.4) density function of the distance between two points chosen at random, independently, and with uniform distribution in a ball of diameter d we obtain:

$$f_\rho(r) = \frac{24 r^2}{d^3} + \frac{12 r^5}{d^6} - \frac{36 r^3}{d^4}. \tag{0.5}$$

Knowing the density function of the distance between two points chosen at random, independently, and with uniform distribution in D we calculate the k-th moment of the distance between two points chosen at random, independently, and with uniform distribution in D which is expressed in the following formula:

$$M_n^k = \frac{O_{n-2} S_n(D)}{(n-1)(n+k) V_n^2(D)} \int_0^d x^{n+k} (1 - F_D(x)) dx \tag{0.6}$$

From 0.5 and 0.6 we calculate the mean of the distance between two points chosen at random, independently, and with uniform distribution in ball.

$$M_3^1 = \frac{9}{16r^4} \int_0^d x^4 \Big[1 - \Big(\frac{x}{d}\Big)^2\Big] dx = \frac{18}{35} d.$$

Let P_1 and P_2 be two points chosen at random, independently and with uniform distribution in D. Denote by $(x_{11}, x_{12}, \cdots, x_{1n})$ the coordinates of P_1 and by $(x_{21}, x_{22}, \cdots, x_{2n})$ the coordinates of P_2. Denote by $(x_1, x_2, \cdots, x_n)$ difference of corresponding coordinates of P_1 and P_2 $(x_i = x_{1i} - x_{2i})$. We define distribution function of difference of two points

$$F_{P_1-P_2}(h) = \frac{\int_{(P_1,P_2 \in D: x_1 \le h_1, x_2 \le h_2, \cdots, x_n \le h_n)} dP_1 dP_2}{V_n^2(D)}$$

We obtain the relationship between distribution function of difference of two points chosen at random, independently, and with uniform distribution in D and the covariogram of D

$$f_{P_1-P_2}(h) = \frac{C(D,h)}{V_n^2(D)}$$

and also we obtain the relationship between density function of $P_1 - P_2$ and orientation depend chord length distribution function

$$f_{P_1-P_2}(h) = \frac{1}{V_n^2(D)} \bigg(V_n(D) - b(D,u) \int_0^t (1 - F_D(s,u)) ds \bigg)$$

We get relationship between covariogram and density function of the distance between two randomly and independently chosen points

$$f_\rho(t) = \frac{1}{2} \int_{S_{n-1}} r^{n-1} \bigg(\frac{C(D,t,u)}{V_n^2(D)} + \frac{C(D,t,u)}{V_n^2(D)} \bigg) du =$$

$$= \frac{t^{n-1}}{V_n^2(D)} \int_{S_{n-1}} C(D,t,u) du \tag{0.7}$$

Using (0.7) we get formula for calculating kinematic measure using covariogram. We know that (see [39])

$$f_\rho(t) = \frac{t^{n-1} K(D,t)}{V_n^2(D)}$$

where K(D,t) is the kinematic measure of all oriented segments of length t that lie entirely inside D (see [1]).

We can calculate k-th moment using the following formula

$$M_n^k = \int_0^d \frac{x^{n+k-1}}{V_n^2(D)} \int_{S^{n-1}} C(D,x,u)dudx$$

In chapter three obtained the relationship between the covariogram of the oblique prism U with base B using the covariogram of its base

$$C(U,t,\omega) = C\Bigg(B, \left(\frac{t\sin(\beta-\theta)}{\sin\beta}\right), \phi\Bigg)(d\sin\beta - t\sin\theta) \tag{0.8}$$

For oblique prism U with base B (not necessarily convex), with prism generator d and angle between prism generator and base β the orientation-dependent chord length distribution given by the following formula

$$F_U(t,\omega) = \begin{cases} 0, & \text{if } t \le 0 \\ \frac{b(B,\phi)(\frac{\sin(\beta-\theta)}{\sin\beta})}{V_2(B)\sin\theta + b(B,\phi)d\sin\beta(\frac{\sin(\beta-\theta)}{\sin\beta})}\Bigg(t\sin\theta + & \\ +\sin\theta\int_0^t (1 - F_B((\frac{u\sin(\beta-\theta)}{\sin\beta}),\phi))du + & \\ +(d\sin\beta - t\sin\theta)F_B((\frac{t\sin(\beta-\theta)}{\sin\beta}),\phi)\Bigg), & \text{if } 0 \le t \le t_{max}(\omega) \\ 1, & \text{if } t \ge t_{max}(\omega) \end{cases} \tag{0.9}$$

Where $t_{max}(\omega)$ is

$$t_{max}(\omega) = \begin{cases} \frac{\sin\beta x_{max}(\phi)}{|\sin(\beta-\theta)|}, & \text{if } \theta \in [0, \arccos\frac{d\sin\beta}{d\sin\beta + x_{max}(\phi)}] \cup \\ & \cup[-\arctan\frac{d\sin\beta}{x_{max}(\phi) - d\sin\beta}, 0] \\ \frac{d\sin\beta}{|\sin\theta|}, & \text{if } [\arccos\frac{d\sin\beta}{d\sin\beta + x_{max}(\phi)}, \pi/2] \cup \\ & \cup[-\pi/2, -\arctan\frac{d\sin\beta}{x_{max}(\phi) - d\sin\beta}] \end{cases} \tag{0.10}$$

when $d\cos\beta < x_{max}(\phi)$ and

$$t_{max}(\omega) = \begin{cases} \frac{\sin\beta x_{max}(\phi)}{|\sin(\beta-\theta)|}, & \text{if } \theta \in [0, \arctan\frac{d\sin\beta}{x_{max}(\phi)+d\sin\beta}] \\ \frac{d\sin\beta}{|\sin\theta|}, & \text{if } \theta \in [\arctan\frac{d\sin\beta}{x_{max}(\phi)+d\sin\beta}, \arctan\frac{d\sin\beta}{d\sin\beta - x_{max}(\phi)}] \\ \frac{\sin\beta x_{max}(\phi)}{|\sin(\theta-\beta)|}, & \text{if } \theta \in [\arctan\frac{d\sin\beta}{d\sin\beta - x_{max}(\phi)}], \pi/2] \cup [-\pi/2, 0] \end{cases} \tag{0.11}$$

when $d\cos\beta > x_{max}(\phi)$.

Let $T \subset R^2$ be a trapezoid with bases a and b and the angle between longer base and legs are ψ_1, ψ_2. Without loss of generality we can assume that $0 < \psi_1 \leq \pi/2$, $\psi_1 \leq \psi_2 < \pi$ and $b < a$. We can translate and rotate trapezoid so that the longer base located in X axis.

$F_T(x,\phi) = 0$ if $x < 0$ and $F_T(x,\phi) = 1$ if $x > x_{max}(\phi)$. Now we discuss the non-trivial cases when $0 < x < x_{max}$. Because this is π periodic function we can assume that k is equal to 0.

(i) For $0 \leq \phi < \psi_1$

$$F_T(x,\phi) = \begin{cases} \frac{x\sin\phi(\sin(\psi_1-\phi)\sin\psi_2+\sin\psi_1\sin(\phi+\psi_2))}{b(T,\phi)\sin\psi_1\sin\psi_2}, & \text{if } 0 \leq x < x_0(\phi) \\ \frac{1}{b(T,\phi)}(b\sin\phi+ & \\ +\frac{(x-x_0(\phi))\sin(\psi_1-\phi)\sin(\psi_2+\phi)}{\sin(\psi_1+\psi_2)}+ & \\ +\frac{x\sin(\phi+\psi_2)\sin\phi}{\sin\psi_2}), & \text{if } x_0(\phi) \leq x < x_{max}(\phi) \end{cases}$$

(ii) For $\psi_1 \leq \phi < \pi - \psi_2$

$$F_T(x,\phi) = \frac{x\sin\phi}{b(T,\phi)}\left(\frac{\sin(\psi_2+\phi)\sin\psi_1+\sin(\phi-\psi_1)\sin\psi_2}{\sin\psi_1\sin\psi_2}\right)$$

(iii) For $\pi - \psi_2 \leq \phi < \pi$

$$F_T(x,\phi) = \begin{cases} \frac{-x\sin\phi(\sin\psi_1\sin(\phi+\psi_2)-\sin(\phi-\psi_1)\sin\psi_2)}{b(T,\phi)\sin\psi_1\sin\psi_2}, & \text{if } 0 \leq x \leq x_0(\phi) \\ \frac{1}{b(T,\phi)}(b\sin\phi- & \\ -\frac{(x-x_0(\phi))sin(\psi_2+\phi)\sin(\phi-\psi_1)}{\sin(\psi_1+\psi_2)}+ & \\ +\frac{x\sin\phi\sin(\phi-\psi_1)}{\sin\psi_1}), & if x_0(\phi) \leq x < x_{max}(\phi) \end{cases}$$

Where $x_0(\phi)$ and $x_1(\phi)$ are lines which has ϕ angle with X-axis, pass through a vertex of trapezoid and make a chord of positive Lebesgue measure,

$$x_0(\phi) = min\chi(l_\phi + y) \text{ and } x_1(\phi) = max\chi(l_\phi + y)$$

Using equations 0.8 and 0.9 we calculate the covariogram and orientation-dependent chord length distribution function of circular oblique prism with base circle of radius r, side d and sides lean over at the base is β (circular oblique prism denote by L_r)

$$C(L_r,t,\omega) = \begin{cases} (d\sin\beta - t\sin\theta)2r^2\arccos\frac{t\sin(\theta-\beta)}{2r\sin\beta}- & \\ -\frac{t\sin(\theta-\beta)}{2\sin\beta}\sqrt{4r^2-\frac{t\sin(\theta-\beta)}{\sin\beta}}, & \text{if } 0 \leq t \leq \chi_{max}(\omega) \\ 0, & \text{otherwise} \end{cases}$$

$$F_{L_r}(t,\omega) = \begin{cases} 0, & \text{if } t < 0 \\ \frac{2\left(\frac{\sin(\beta-\theta)}{\sin\beta}\right)}{\pi r\sin\theta+2d\sin(\beta-\theta)}\left(d\sin\beta-\right. & \\ -(d\sin\beta-\frac{3t\sin\theta)}{2})\left(\sqrt{1-\left(\frac{t\sin(\beta-\theta)}{2r\sin\beta}\right)^2}\right)+ & \\ +\frac{r\sin\theta\sin\beta}{\sin(\theta-\beta)}(\arcsin(\frac{t\sin(\beta-\theta)}{2r\sin\beta})) & \text{if } 0 \leq t < \chi_{max}(\omega) \\ 1, & \text{if } t \geq \chi_{max}(\omega) \end{cases}$$

Using equations 0.8 and 0.9 we calculate the covariogram and orientation-dependent chord length distribution function of an elliptical oblique prism with with prism generator d, the angle with prism generator and base is β and base as an ellipse with semi-major axes a and b (Denote by L_e an elliptical oblique prism and by $E_{a,b}$ its base)

$$C(L_e,t,\omega)=2ab\Big(\frac{\pi}{2}-\frac{t\sin(\beta-\theta)}{\chi_{max}(\phi)\sin\beta}\sqrt{1-\frac{t^2\sin^2(\beta-\theta)}{\chi_{max}(\phi)}\sin^2\beta}-$$
$$-\arcsin\frac{t\sin(\beta-\theta)}{\chi_{max}(\phi)\sin\beta}\Big)\Big(d\sin\beta-t\sin\theta\Big)$$

$$F_{L_e}(t,\omega)=\begin{cases}0, & \text{if } t<0\\ \frac{b(E_{a,b},\phi)\left(\frac{\sin(\beta-\theta)}{\sin\beta}\right)}{\pi ab\sin\theta+b(E_{a,b},\phi)d\sin(\beta-\theta)}\Big(d\sin\beta- & \\ -(d\sin\beta-\frac{3t\sin\theta)}{2})\left(\sqrt{1-\left(\frac{t\sin(\beta-\theta)}{\chi_{max}(\phi)\sin\beta}\right)^2}\right)+ & \\ +\frac{\chi_{max}(\phi)\sin\theta\sin\beta}{2\sin(\theta-\beta)}(\arcsin(\frac{t\sin(\beta-\theta)}{\chi_{max}(\phi)\sin\beta})) & \text{if } 0\le t<\chi_{max}(\omega)\\ 1, & \text{if } t\ge\chi_{max}(\omega)\end{cases}$$

and $b(E_{a,b},\phi)$ is equal to

$$b(E_{a,b},\phi)=\sqrt{a^2\sin^2\phi+b^2\cos^2\phi}$$

Let L_Δ denote an oblique prism with triangular base Δ. We consider the side of Δ that lies on the X axes. Let a be the length of that side, and ψ_1 and ψ_2 be the corresponding adjacent angles.equations 0.8 and 0.9 we calculate the covariogram and orientation-dependent chord length distribution function of L_Δ

$$C(L_\Delta,t,\phi)=\begin{cases}S_\Delta\left(1-\frac{t\sin(\beta-\theta)}{\sin\beta\chi_{max}(\phi)}\right)^2\Big(s\sin\beta-t\sin\theta\Big), & \text{if } 0\le t<\chi_{max}(\omega)\\ 0, & \text{otherwise}\end{cases}$$

where S_Δ is the area of the triangle Δ.

$$F_{L_\Delta}(t,\omega) = \begin{cases} 0, & \text{if } t \leq 0 \\ \frac{tb(B,\phi)\left(\frac{\sin(\beta-\theta)}{\sin\beta}\right)}{S_\Delta \sin\theta + b(B,\phi) d \sin\beta\left(\frac{2\sin(\beta-\theta)}{\sin\beta}\right)} \\ \left(2\sin\theta - \frac{3tsin\theta\sin(\beta-\theta)}{2\sin\beta\chi_{max}(\phi)} + \frac{d\sin(\beta-\theta)}{\chi_{max}(\phi)}\right), & \text{if } 0 \leq t \leq t_{max}(\omega) \\ 1, & \text{if } t \geq t_{max}(\omega) \end{cases}$$

In desertation it also calculated the orientation-dependent chord length distribution function for an oblique prism with trapezoid base.

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